Journal of Engineering Physics and Thermophysics, Vol. 72, No. 4, 1999

HYDRODYNAMIC DRAG OF A HYDROSOL SPHEROID-SHAPED PARTICLE HEATED BY INTERNAL HEAT SOURCES AT SMALL REYNOLDS NUMBERS

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UDC 533.72

The problem of flow past a nonuniformly heated hydrosol spheroid-shaped particle is solved. An analytical expression is obtained for the hydrodynamic force acting on the particle..

The motion of heated particles in viscous liquid and gas media was considered in a number of works [1-5]. The term heated is understood to refer to a particle the mean temperature of the surface of which considerably exceeds in value the temperature of the environment. The surface of the particle is heated due to the presence of internal heat sources whose appearance can be attributed, for example, to the occurrence of a chemical reaction proceeding in its volume, to the process of the radioactive decay of the substance of the particle, etc. The resulting rise in the temperature of the particle surface can exert a substantial effect on the thermophysical characteristics of the environment and thereby considerably affect the distribution velocity and pressure fields in the vicinity of the particle.

However, up to now the motion of heated particles in viscous gas and liquid media was considered only for particles of spherical shape [1-5]. Many particles that are met in industrial equipment and in nature have a shape of the surface differing from a spherical one, e.g., spheroidal. In the present work, an analytical expression has been obtained in the Stokes approximation for a hydrodynamic force that acts on a spheroidal particle of density q_p heated by internal heat sources.

We will consider the motion of a nonuniformly heated hydrosol particle with the shape of an oblate spheroid, at small Reynolds and Peclet numbers along the symmetry axis of the spheroid under the action of a certain force (gravitational, magnetic, electrophoretic, etc.) If we change to the coordinate system connected with the particle, then the problem is reduced in essence to the problem of flow of a plane-parallel stream of liquid with velocity $\mathbf{U}_\infty$ ($\mathbf{U}_\infty \parallel OZ$) past a nonuniformly heated stationary hydrosol particle having the form of an oblate (prolate) spheroid.

Of all the parameters of the transfer of liquid, only the coefficient of dynamic viscosity depends strongly on temperature [6]. In view of a weak dependence of the density and thermal conductivity of the substance of the particle and carrier medium on temperature, we will consider them to be constant. To take account of the temperature dependence of viscosity we shall avail ourselves of the expression [3, 6]

$$\frac{\mu_{\text{liq}}}{\mu_\infty} = \exp\left[-A\left(\frac{T_{\text{liq}}}{T_\infty} - 1\right)\right]. \tag{1}$$

The description of the flow past a spheroid is made in a spheroidal coordinate system (ε, η, φ) with the origin fixed at the center of a hydrosol particle. They are connected with the Cartesian coordinates by the following relations [7]

$$x = c \sinh \varepsilon \sin \eta \cos \varphi\,, \quad y = c \sinh \varepsilon \sin \eta \sin \varphi\,, \quad z = c \cosh \varepsilon \cos \eta\,, \tag{2}$$

Belarusian State University, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 4, pp. 651-655, July-August, 1999. Original article submitted January 20, 1998; revision submitted August 19, 1998.

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$$x = c\cosh\varepsilon\sin\eta\cos\varphi\,,\quad y = c\cosh\varepsilon\sin\eta\sin\varphi\,,\quad z = c\sinh\varepsilon\cos\eta\,, \tag{3}$$

where $c = \sqrt{b^2 - a^2}$ in the case of a prolate spheroid ($a < b$, formula (2)) and $c = \sqrt{a^2 - b^2}$ in the case of an oblate spheroid ($a > b$, formula (3)); a and b are the semiaxes of a spheroid. Moreover, the position of the Cartesian coordinate system is fixed relative to the particle so that the z axis could coincide with the symmetry axis of the spheroid. The thermal conductivity coefficient of the spheroid λ_p in magnitude is much in excess of the thermal conductivity coefficient of the surrounding liquid λ_{liq}.

At small Reynolds and Peclet numbers in a quasistationary approximation the distribution of velocity V_{liq}, pressure P_{liq}, temperature T_{liq}, and of T_p is described by the following system of equations [8]

$$\frac{\partial}{\partial x_k} P_{liq} = \frac{\partial}{\partial x_j}\left\{\mu_{liq}\left(\frac{\partial V_k^{liq}}{\partial x_j} + \frac{\partial V_j^{liq}}{\partial x_k}\right)\right\},\quad \mathrm{div}\,\mathbf{V}_{liq} = 0\,; \tag{4}$$

$$\mathrm{div}\,(\lambda_{liq}\nabla T_{liq}) = 0\,,\quad \mathrm{div}\,(\lambda_p\nabla T_p) = -\,q_p, \tag{5}$$

In solving the system of equations (4)-(5) we take into account the following boundary conditions

$$\mathbf{V}_{liq} = 0\,,\quad T_{liq} = T_p\,,\quad \lambda_{liq}\frac{\partial T_{liq}}{\partial\varepsilon} = \lambda_p\frac{\partial T_p}{\partial\varepsilon}\quad \text{for}\quad \varepsilon = \varepsilon_0\,, \tag{6}$$

$$\mathbf{V}_{liq} = U_\infty\cos\eta\,\mathbf{e}_\varepsilon - U_\infty\sin\eta\,\mathbf{e}_\eta\,,\quad T_{liq} = T_\infty\,,\quad P_{liq} = P_\infty\quad \text{for}\quad \varepsilon = \infty\,.$$

On the surface of the particle these boundary conditions (6) take account of: the condition of sticking for velocity, the equality of temperatures, and the continuity of heat fluxes. The coordinate surface with an ε value equal to ε_0 corresponds to the surface of the particle.

To find the force that acts on a solid heated spheroidal particle, it is necessary to know the temperature fields outside and inside of it. Integrating Eq. (5) with corresponding boundary conditions, we obtain

$$t_{liq} = 1 + \gamma\arctan\lambda\,, \tag{7}$$

$$t_p = B + \frac{\lambda_{liq}}{\lambda_p}\gamma\arctan\lambda + \int_{\lambda_0}^{\lambda}\arctan\lambda\, f_0\,d\lambda - \arctan\lambda\int_{\lambda_0}^{\lambda} f_0\,d\lambda\,, \tag{8}$$

where $t = T/T_0$; $\lambda = \sinh\varepsilon$; $\lambda_0 = \sinh\varepsilon_0$; $B = 1 - \arctan\lambda_0\gamma(1 - \lambda_{liq}/\lambda_p)$; $\gamma = t_s - 1$ is the dimensionless parameter that characterizes the heating of the surface of the particle; $f_0 = -\frac{1}{2\lambda_p T_\infty}\int_{-1}^{1} c^2(\lambda^2 + x^2)q_p dx$, $x = \cos\eta$; $t_s = T_s/T_\infty$, T_s is the mean spheroid-surface temperature determined from the relation

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi c\lambda_{liq}T_\infty}\int_V q_p dV\,. \tag{9}$$

Integration of (9) extends over the entire volume of the particle.

From formula (7) it is seen that the temperature in the liquid changes with distance from the surface of the particle. Consequently, viscosity is also a function of distance, $\mu_{liq} = \mu_{liq}(\lambda)$. Since the viscosity depends only on the radial coordinate λ, a solution of the system of equations (4) is found by the separation-of-variables technique. Here the components of mass velocity must be sought in the form

$$V_\varepsilon^{liq}(\varepsilon,\eta) = \frac{U_\infty}{c\cosh\varepsilon H_\varepsilon}G(\varepsilon)\cos\eta\,,\quad V_\eta^{liq}(\varepsilon,\eta) = -\frac{U_\infty}{cH_\varepsilon}g(\varepsilon)\sin\eta\,,$$

where $H_\varepsilon = c\sqrt{\cosh^2\varepsilon - \sin^2\eta}$; $G(\varepsilon)$ and $g(\varepsilon)$ are arbitrary functions that depend on the radial coordinate ε.

As a result, expressions were found for velocity and pressure fields. Integrating the stress tensor over the surface of a spheroid [7], we obtain an expression for the hydrodynamic resistance of a heated hydrosol particle that has the shape of an oblate spheroid:

$$\mathrm{F} = 4\pi\mu_\infty U_\infty\, c f_\mu \exp\left[-A\gamma \arctan\lambda_0\right\}\mathrm{n}_z, \tag{10}$$

where $f_\mu = G_1'/(G_1G_2' - G_2G_1')$; G_1' and G_2' are the first derivatives of the functions G_1 and G_2 with respect to λ;

$$G_1 = \sum_{n=0}^{\infty}\frac{C_n^{(1)}}{(n+3)\lambda^{n+3}};\quad G_2 = \sum_{n=0}^{\infty}\frac{C_n^{(2)}}{(n+1)\lambda^{n+1}} + \beta\sum_{n=0}^{\infty}\frac{C_n^{(1)}}{(n+3)^2\lambda^{n+3}}\left[(n+3)\ln\frac{\lambda_0}{\lambda} - 1\right];$$

n_z is the unit vector in the direction of the z axis; $\lambda = b/c$.

The functions G_1, G_2, G_1', and G_2', which enter into the expression for the force (10), are calculated for $\lambda = \lambda_0$. The values of the coefficients $C_n^{(1)}$ and $C_n^{(2)}$ are determined from the recurrent relations:

$$C_n^{(1)} = \frac{1}{n(n+5)}\left\{\frac{4}{5}(5n+13)C_{n-2}^{(1)} + 12\sum_{k=0}^{[(n-2)/2]}(-1)^k \times\right.$$

$$\times\frac{(n-2k+1)(n-2k+2)}{(2k+1)(2k+3)(2k+5)}C_{n-2k-2}^{(1)} -$$

$$-12\sum_{k=0}^{[(n-4)/2]}(-1)^k\frac{(4k+8)(n-2k)+(2k+3)(2k+5)}{(2k+3)(2k+5)(2k+7)}C_{n-2k-4}^{(1)} -$$

$$-\gamma_0\left[(n+1)C_{n-1}^{(1)} + 4C_{n-3}^{(1)} - 3\sum_{k=0}^{[(n-3)/2]}(-1)^k\frac{n-2k+1}{(2k+3)(2k+5)}C_{n-2k-3}^{(1)} -\right.$$

$$\left.\left.-6\sum_{k=0}^{[(n-5)/2]}(-1)^k\frac{(k+3)(4k+9)}{(2k+5)(2k+7)}C_{n-2k-5}^{(1)}\right]\right\}, \tag{11}$$

$$C_n^{(2)} = \frac{1}{(n+3)(n-2)}\left\{\frac{4}{5}(5n+3)C_{n-2}^{(2)} + 12\sum_{k=0}^{[(n-2)/2]}\times\right.$$

$$\times(-1)^k\frac{(n-2k)(n-2k-1)}{(2k+1)(2k+3)(2k+5)}C_{n-2k-2}^{(2)} -$$

$$-12\sum_{k=0}^{[(n-4)/2]}(-1)^k\frac{(4k+9)(n-2k-2)+(2k+3)(2k+5)}{(2k+3)(2k+5)(2k+7)}C_{n-2k-4}^{(2)} -$$

$$-\gamma_0\left[(n-1)C_{n-1}^{(2)} + 4C_{n-3}^{(2)} - 3\sum_{k=0}^{[(n-3)/2]}(-1)^k\frac{n-2k-1}{(2k+3)(2k+5)}C_{n-2k-3}^{(2)} -\right.$$

$$\left.-6\sum_{k=0}^{[(n-5)/2]}(-1)^k\frac{(k+3)(4k+9)}{(2k+5)(2k+7)}C_{n-2k-5}^{(2)}\right] + \beta\left[4C_{n-4}^{(1)} + (2n+1)C_{n-2}^{(1)} - 12\sum_{k=0}^{[(n-6)/2]}(-1)^k\times\right.$$

$$\times\frac{4k+9}{(2k+3)(2k+5)(2k+7)}C_{n-2k-6}^{(1)} - \gamma_0\left\{C_{n-3}^{(1)} - 3\sum_{k=0}^{[(n-5)/2]}(-1)^k\frac{1}{(2k+3)(2k+5)}C_{n-2k-5}^{(1)}\right\} +$$

TABLE 1. Dependence of f_μ on the Mean Temperature of the Surface of the Spheroid and Semiaxes Ratio a/b

a/b	T_s, K			
	293	313	331	353
1.2	2.066	2.023	1.918	1.804
1.4	0.978	0.649	0.467	0.373

$$+ 12 \sum_{k=0}^{[(n-4)/2]} (-1)^k \frac{2n-4k-1}{(2k+1)(2k+3)(2k+5)} C_{n-2k-4}^{(1)} \Bigg] + 6 \sum_{k=0}^{[n/2]} (-1)^k (1-k)\, a_{n-2k} \Bigg\} . \tag{12}$$

In calculation of the coefficients $C_n^{(1)}$ and $C_n^{(2)}$ from the above recurrent formulas it is necessary to take into account that $C_0^{(1)} = 1$, $C_2^{(2)} = 1$, $C_0^{(2)} = 1$, $C_1^{(2)} = -\frac{3}{2}a_1$, $\beta = -\frac{3}{10}(a_1\gamma_0 + 4a_2)$, $a_{n+1} = \frac{1}{n+1}[\gamma_0 a_n - (n-1)a_{n-1}]$ $(n \geq 1)$, $\gamma_0 = A\gamma$, $a_0 = 1$, C_n for $n < 0$ are equal to zero. The integer part of the number $k/2$ is denoted by $[\frac{k}{2}]$. Recurrent formula (11) is valid for $n \geq 1$ and formula (12) for $n \geq 3$.

To obtain an expression for the hydrodynamic resistance of a prolate spheroid, it is necessary in (10) to replace λ by $i\lambda$ and c by ic (i is an imaginary unit).

Thus, formula (10) makes it possible to evaluate the hydrodynamic force acting on a nonuniformly heated spheroid-shaped particle.

The results of calculations of the dependence of f_μ on the mean temperature of the surface of the spheroid and on the semiaxes ratio (a/b) are listed in Table 1 for $T_\infty = 293$ K and $A = 6.095$ for a mercury drop suspended in water.

As an example, we will consider the motion of a spheroidal particle in the gravity field. The particle, which is impinged by gravity in a viscous liquid, ultimately begins to move with a constant velocity at which the gravity force is balanced by hydrodynamic forces.

The gravity force acting on the particle with account taken of the buoyancy force is equal to

$$\mathbf{F} = (\rho_p - \rho_{liq})\, g \frac{4}{3}\pi a^2 b . \tag{13}$$

Equating (10) and (13), we obtain the velocity of the steady incidence of the nonuniformly heated spheroid-shaped particle:

$$\mathbf{U}_\infty = (\rho_p - \rho_{liq})\, g \frac{a^2 b}{3\mu_\infty c} \frac{G_1 G_2' - G_2 G_1'}{G_1'} \exp\left\{A\gamma \arctan \lambda_0\right\} \mathbf{n}_z . \tag{14}$$

In the limit of $\gamma \to 0$ (small temperature differences in the vicinity of the spheroid) the formulas obtained transform into well-known expressions [7].

NOTATION

$\mu_\infty = \mu_{liq}(T_\infty)$, T_∞, temperature at a distance from the spheroid; A, constant; T_{liq}, T_p, ρ_{liq}, and ρ_p, temperature and density of the liquid and particle; $\mathbf{V}_{liq}$, mass velocity; $q_p(\varepsilon, \eta)$, density of the heat sources inside the particle as a function of the spheroidal coordinates ε and η $(0 \leq \eta \leq \pi)$; $\mathbf{U}_\infty$, velocity of the plane-parallel flow of liquid past the spheroid ($\mathbf{U}_\infty \parallel OZ$); λ_{liq} and λ_p, thermal conductivity coefficients of the liquid and particle; μ_{liq}, coefficient of dynamic viscosity; P_∞ and T_∞, nonperturbed pressure and temperature in the liquid; g, free-fall acceleration. Subscripts: liq and p refer to the liquid and particle; ∞ denotes values of physical quantities far from the particle (at infinity).

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